

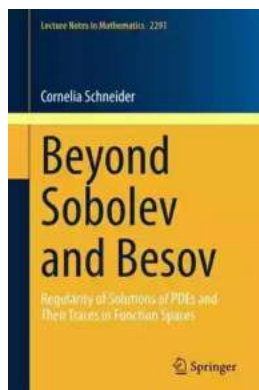
Beyond Sobolev and Besov: Unraveling the Depths of Function Spaces

When it comes to analyzing functions, mathematicians have a plethora of powerful tools at their disposal. These tools help them understand the properties and behaviors of various mathematical functions, paving the way for advancements in numerous fields, from physics and engineering to computer science and finance. Two essential function spaces that have played a significant role in this endeavor are Sobolev and Besov spaces.

Sobolev Spaces: The Path to Smoothness

Sobolev spaces were introduced by a Russian mathematician, Sergei Lvovich Sobolev, in the mid-20th century. These spaces provide a framework for studying differentiability of functions and solving partial differential equations.

At its core, Sobolev spaces extend the concept of smoothness in functions. While most functions encountered in practical applications are not infinitely differentiable, Sobolev spaces provide a way to quantify and measure their level of differentiability. Functions within these spaces possess certain regularity properties, which are useful in many mathematical models.



Beyond Sobolev and Besov: Regularity of Solutions of PDEs and Their Traces in Function Spaces (Lecture Notes in Mathematics Book 2291)

by Patrick Suppes (1st ed. 2021 Edition, Kindle Edition)

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The key idea behind Sobolev spaces lies in the usage of weak derivatives, which extend the notion of classical derivatives to functions with less regularity. By allowing functions to have generalized or weaker derivatives, Sobolev spaces capture a wider array of functions, from smooth ones to those with singularities or discontinuities.

The Sobolev space is denoted by $W^{k,p}(\Omega)$, where k denotes the highest order of differentiability, and p represents the integrability of the function. These spaces find applications in fields such as control theory, image processing, and fluid dynamics.

Besov Spaces: The Quest for Local Regularity

Besov spaces, named after the Russian mathematician Vladimir Mikhailovich Besov, arose as extensions of Sobolev spaces with the aim of capturing local regularity properties of functions.

In many mathematical problems, it is crucial to examine not only the overall behavior of a function but also its behavior at small scales or localized regions. This is where Besov spaces shine. They provide a refined understanding of the localized regularity of a function by measuring its smoothness in different regions.

The key concept in Besov spaces is the usage of mixed smoothness and integrability parameters. A function belongs to a Besov space if it satisfies certain conditions related to its smoothness and growth properties. Besov spaces encompass a wide range of functions, from smooth ones to those with oscillatory behaviors or localized singularities.

Like Sobolev spaces, Besov spaces have found applications in numerous domains, including harmonic analysis, image reconstruction, and signal processing. These spaces enable mathematicians and scientists to analyze and manipulate functions with intricate properties at different scales.

Beyond Sobolev and Besov: Exploring New Frontiers

While Sobolev and Besov spaces have been vital for understanding function spaces and their properties, recent developments have pushed the boundaries of analysis to new realms.

One such extension is the Triebel-Lizorkin spaces, which combine elements from both Sobolev and Besov spaces. These spaces allow for a fine-tuned control of both global and local regularity properties of functions. Triebel-Lizorkin spaces have proven to be highly versatile in studying various problems in analysis, partial differential equations, and function approximation.

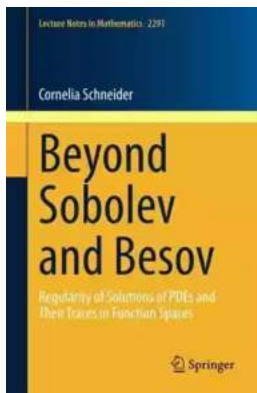
Another frontier lies in Morrey spaces, named after the American mathematician Charles B. Morrey Jr. These spaces extend the notion of Sobolev spaces to study the Hessian matrix, capturing information about the second-order derivatives of functions. Morrey spaces have significant applications in the study of elliptic and parabolic partial differential equations, as well as variational problems.

Furthermore, the recent developments in function spaces have witnessed the emergence of Hörmander spaces, Coifman-Meyer spaces, and many more. These spaces offer specialized frameworks to analyze specific properties, such as high-frequency oscillations, sparse representations, or localized behaviors.

Function spaces form the backbone of mathematical analysis, allowing the study of a wide range of functions and their behaviors. Sobolev and Besov spaces have served as crucial pillars in this area, providing mathematicians and scientists with

powerful tools to analyze and work with functions of varying smoothness. However, these spaces are just the beginning of a vast and ever-evolving landscape.

New frontiers, such as Triebel-Lizorkin spaces, Morrey spaces, and other specialized function spaces, reveal unprecedented depths and potential for further exploration. As mathematicians delve deeper into these function spaces, new insights and applications are bound to emerge, revolutionizing our understanding of functions and their intricate properties.



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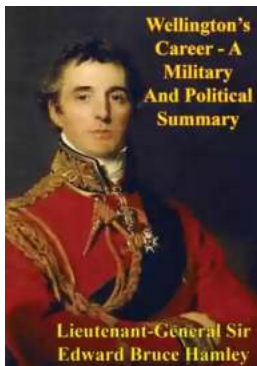
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This book investigates the close relation between quite sophisticated function spaces, the regularity of solutions of partial differential equations (PDEs) in these spaces and the link with the numerical solution of such PDEs. It consists of three parts. Part I, the , provides a quick guide to function spaces and the general concepts needed. Part II is the heart of the monograph and deals with the regularity of solutions in Besov and fractional Sobolev spaces. In particular, it studies regularity estimates of PDEs of elliptic, parabolic and hyperbolic type on non smooth domains. Linear as well as nonlinear equations are considered and

special attention is paid to PDEs of parabolic type. For the classes of PDEs investigated a justification is given for the use of adaptive numerical schemes. Finally, the last part has a slightly different focus and is concerned with traces in several function spaces such as Besov– and Triebel–Lizorkin spaces, but also in quite general smoothness Morrey spaces.

The book is aimed at researchers and graduate students working in regularity theory of PDEs and function spaces, who are looking for a comprehensive treatment of the above listed topics.



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